

STATEMENT REPORT

under the procedure for acquisition of the educational and scientific degree “Doctor”

by candidate **Martin Jordanov (Yordanov) Minchev**,

of the PhD Thesis entitled: “Functionals of Levy processes and their applications”,

In the Scientific field: **4. Natural Sciences, Mathematics and Informatics**

Professional field: **4.5. Mathematics**

Doctoral program “Probability theory and mathematical statistics”,

Department „Probability, operational research and statistics”,

Faculty of Mathematics and Informatics (FMI), Sofia University “St. Kl. Ohridski” (SU),

The statement report has been prepared by: **Professor PhD Pavlina Kalcheva Jordanova (Yordanova)**, deptment “Economics and mathematical modelling”, Faculty of Mathematics and Computer Science, Konstantin Preslavsky University of Shumen as a member of the scientific jury for the defense of this PhD thesis according to Order № RD 38-200/26.04.2024 of the Rector of the Sofia University.

1. General characteristics of the dissertation thesis and the presented materials

The presented dissertation contains 175 pages inclusively: introduction (Chapter I), four theoretical chapters (II-V), conclusion, two appendices and 173 titles in bibliography.

2. Short CV and personal impressions of the candidate

In 2013 Martin Minchev completed his secondary education at Sofia Mathematical High School "Paisiy Hilendarski". In the 2014/2015 academic year, he studied "Mathematics and Physics" at the Lycée "Louis the Great" in Paris, France. He completed his studies as a bachelor in "Applied Mathematics" at Sofia University "St. Kliment Ohridski" (SU) in 2018. In the 2018/2019 academic year he studied and obtained a master's degree in "Probability and Stochastic Models" at the Faculty of Sciences and Engineering at Sorbonne University, Paris, France. Since 2015 he works as a part-time lecturer at SU, where he is an RI researcher since March 2023. He participates in numerous scientific projects with the main idea of supporting young scientists. My personal impressions are that Martin Minchev is ambitious.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

The content of the candidate's thesis shows that the candidate possesses the necessary knowledge and skills to obtain the scientific degree “doctor”. In more detail, this conclusion is supported by the following joint contributions with his supervisor. In Chapter 21 they obtain new results about $I_{\xi}(\infty) = \int_0^{\infty} e^{-\xi_s} ds$, where ξ is a Levy process. Under the assumption that the Levy process ξ drifts to minus infinity, and the tail of its cumulative distribution function (c.d.f.) of ξ_1 is regularly varying with parameter $\alpha > 1$, they show that when t increases unboundedly the probability density function of $I_{\xi}(t) = \int_0^t e^{-\xi_s} ds$ in y , normalized with y^{α} , $\alpha \in (0,1)$, multiplied by the tail of the c.d.f. of ξ_1 converges weakly to a

finite positive measure, concentrated on $(0, \infty)$. In Theorem 23.1 (the first their contribution) and its corollaries, they give a semi-explicit analytic description of the c.d.f. of this measure and, by using Tauberian theorems, present the asymptotic behavior of the expectation of “ a ” powers of $I_{\xi}(t)$. Moreover, they obtain novel analytic properties for the limit measure, such as the existence and smoothness of its density. Later on, dropping the requirement for regular variation of the upper tail of the c.d.f. of ξ_1 , and finiteness of its expectation, they obtain upper bounds for the decay of the expectation of different functions of $I_{\xi}(t)$.

Theorem 23.5 is the second main result in this thesis. Here the author computes the Mellin’s transform of the c.d.f. of the finite measure which appear in Theorem 23.1. This allows him to deduce analytic properties for the density of the limiting measure and to obtain a product factorisation of the random variable behind the normed version of this measure in terms of well-known independent random variables, namely the classical exponential functional of the subordinator related to the Wiener-Hopf factor $\phi_{0,+}$ and the remainder term pertaining to $\phi_{0,-}$.

In theorem 23.6 (the third their main result) they show that the rate of decay depends on the finiteness of an explicit integral criterion. In the proofs of these results the author uses different functional analytic properties of Bernstein-gamma functions, recently introduced and investigated by his supervisor, which seems to be a key tool for the study of both classical exponential functionals and exponential functionals on deterministic horizon. By using Mellin’s transform of the classical exponential functional and by Mellin’s inversion one can obtain information about the exponential functional at exponential time and in turn by computing the Laplace transforms to draw conclusions regarding $I_{\xi}(t)$. The results on the derivatives of bivariate Bernstein-gamma functions allow Martin Minchev to derive the weak convergence of the measures defined in (21.2). More results on their finiteness are presented in Section 24. In Theorem 24.3 it is shown that bivariate Bernstein-gamma functions related to the Wiener-Hopf factors of transient Levy processes are n times differentiable at zero in the first variable if and only if an explicit integral criterion is finite. Lemma 24.1 links the derivatives of these functions to the convolutions and the derivatives of the q -potential measures of the underlying Levy process. Corollary 24.4 presents universal and seemingly quite handy estimates for the derivatives of the Mellin’s transforms of the classical exponential functionals and their decay along complex lines. Some auxiliary results are proven in Section 27.

4. Approbation of the results

For the procedure, one publication from 2023 is presented, in the journal *Bernoulli*, which according to *Web of Science (WoS)* has an impact factor (IF) of 1.5 for the last published year 2022 and is from the Q2 quartile in the field of “Statistics and Probability”. This scientific work meets the minimum national requirements (under Art. 2b, para. 2 and 3 of ADASRB*) and, accordingly, the additional requirements of SU “St. Kliment Ohridski” for the acquisition of an educational and scientific degree “doctor” in the scientific field and professional direction of the procedure.

According to the submitted declaration of co-authorship, the contribution of Martin Minchev and his scientific supervisor is equal.

The results of the dissertation are also presented through:

- *posters at an international conference in Lisbon, Portugal, and another one in Manchester, Great Britain;*
- *report on Mathematics days in Sofia 2023,*
- *report of the FMI Spring Scientific Session,*
- *report of a scientific seminar in Veliko Tarnovo.*

The results of the candidate and his scientific supervisor have been well received by the international scientific community in the area of the procedure, as evidenced by the seven citations presented:

- *one of which is in a WoS Q1 journal, in the field of "Mathematics", with a WoS IF for 2022, 1.8.*
- *another one is found in a WoS Q2 journal, in the "Statistics and Probabilities" direction, with a WoS IF for 2022, 1.5.*
- *another one is in a WoS Q3 journal, in the "Statistics and Probabilities" direction, with a WoS IF for 2022, 1.4.*
- *another one is in a journal with Q4 of WoS, in the direction "Statistics and Probabilities", with an IF of WoS for 2022, 0.5 and*
- *another one is in a PhD dissertation at Cornell University, USA.*

The results presented by the candidate in the dissertation work and in the article to it do not repeat such from previous procedures for acquiring a scientific degree and academic position in which the candidate have participated.

I do not find any plagiarism in the submitted dissertation and scientific work under this procedure.

5. Qualities of the abstract

The abstract correctly and fully reflects the results and content of the dissertation work and meets all the requirements for its preparation. I believe that the contributions claimed in the dissertation are indeed such.

6. Critical notes and recommendations

Critical remarks addressed to the PhD thesis and accompanying papers are given in relation to: staging; analyzes and summaries; methodological level; accuracy and completeness of the results; literary awareness.

In the submitted dissertation, the candidate's contributions are not clearly distinguished from the contributions of his supervisor. The work was not presented with a report at an international conference outside the country. The author should understand the benefits of giving talks at international scientific conferences outside the country, and to present more frequently there his papers. And last but not least, the presented results certainly find application when working with real data. Considering not only the scientific, but also the applied nature of the direction in which the candidate is defending, I would be happy to see more examples of conclusions from modeling real data with the proposed techniques.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, **I confirm** that the presented PhD thesis and the scientific publication to it, as well as the quality and originality of the results and achievements presented inside, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field 4. Natural science, mathematics and informatics, Professional field, 4.5 Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, **I recommend** the scientific jury to award **Martin Jordanov (Yordanov) Minchev**, the educational and scientific degree "Doctor" in the Scientific field 4. Natural science, mathematics and informatics, Professional field, 4.5 Mathematics.

Date: 18.06.2024

Signature:

/Pavlina Jordanova, Professor, Dr./